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MATDIP301

Third Semester B.E. Degree Examination, Feb./Mar. 2022

**Advanced Mathematics – I**

Time: 3 hrs.

Max. Marks: 100

**Note: Answer any FIVE full questions.**

- 1 a. Express the complex number  $\frac{2 - i\sqrt{3}}{\sqrt{3} + i}$  in the form of  $x + iy$ . (06 Marks)
- b. Express  $\sqrt{3} + i$  in the polar form and hence find modulus and amplitude. (07 Marks)
- c. Prove that  $\left(\frac{\cos\theta + i\sin\theta}{\sin\theta + i\cos\theta}\right)^4 = \cos 8\theta + i\sin 8\theta$ . (07 Marks)
- 2 a. Find the  $n^{\text{th}}$  derivative of  $e^{ax} \sin(bx + c)$ . (06 Marks)
- b. If  $y = \sin^{-1} x$  then prove that  $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0$ . (07 Marks)
- c. Find the  $n^{\text{th}}$  derivative of  $\frac{x + 3}{(x + 2)(2x + 3)}$ . (07 Marks)
- 3 a. Find the angle between the curves  $r = \sin\theta + \cos\theta$  and  $r = 2\sin\theta$ . (06 Marks)
- b. Find the pedal equation for the curve  $r^2 = a^2 \cos 2\theta$ . (07 Marks)
- c. Using Maclaurin's series expand  $y = \tan x$  upto the term containing  $x^5$ . (07 Marks)
- 4 a. If  $u = \sin^{-1}\left(\frac{x^2 + y^2}{x + y}\right)$ , prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u$ . (06 Marks)
- b. If  $u = f(xz, y/z)$ , prove that  $x \frac{\partial u}{\partial x} - y \frac{\partial u}{\partial y} - z \frac{\partial u}{\partial z} = 0$ . (07 Marks)
- c. If  $x + y + z = u$ ,  $y + z = v$  and  $z = uvw$  then find the value of  $\frac{\partial(x, y, z)}{\partial(u, v, w)}$ . (07 Marks)
- 5 a. Obtain the reduction formula for  $\int \sin^n x dx$ , where  $n$  is a positive integer. (06 Marks)
- b. Evaluate  $\int_0^{\pi/2} \sin^3 \theta \cos^7 \theta d\theta$ . (07 Marks)
- c. Evaluate  $\int_0^a \int_0^{\sqrt{a^2 - x^2}} x^2 y^2 dy dx$ . (07 Marks)
- 6 a. Evaluate  $\int_0^1 \int_0^2 \int_0^3 xyz dz dy dx$ . (06 Marks)
- b. Define Beta function and prove that  $\beta(m, n) = \beta(n, m)$ . (07 Marks)
- c. Using beta and gamma functions evaluate  $\int_0^{\pi/2} \sin^4 \theta \cos^5 \theta d\theta$ . (07 Marks)



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- 7 a. Solve  $\frac{dy}{dx} = \frac{y}{x} + \sin \frac{y}{x}$ . (06 Marks)
- b. Solve  $(1 + y^2)dx = (\tan^{-1} y - x)dy$ . (07 Marks)
- c. Solve  $(1 + e^{x/y})dx + e^{x/y}(1 - x/y)dy = 0$ . (07 Marks)
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- 8 a. Solve  $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x} + 4$ . (06 Marks)
- b. Solve  $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x x^2$ . (07 Marks)
- c. Solve  $(D^2 + D + 1)y = x^2 + x + 1$ . (07 Marks)

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